Quantitative Evaluation of the Linear Reduced-order Model based on PIV Data for Future Flow Control

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1 Introduction

Recently, active flow control devices such as dielectric barrier discharge plasma actuators have attracted a lot of attention. Many studies have shown that they are effective to improve the aerodynamic performance of airfoils. The control performance of the active flow control devices is affected by the control input, thus it is important to determine the control input based on the state of flow field around the airfoil [1]. This motivates us to construct an optimal feedback flow control system using a model which estimates the state of the flow field.

In the past, the authors have studied the construction of a linear reduced-order model of flow fields around an airfoil based on experimental data [2]. The time-resolved particle image velocimetry (PIV) measurement was conducted at the wind tunnel test and unsteady data of velocity field around a NACA0015 airfoil were obtained at the chord Reynolds number of $6.4 \times 10^4$ and the angle of attack from 11º to 20º. The proper orthogonal decomposition (POD) was applied to the experimental data and orthogonal basis that express the data with the utmost efficiency (POD modes) were acquired. Subsequently, the low-dimensional description of the flow fields was obtained by truncating high-order POD modes and the computational cost for the estimation by the model was reduced drastically. The linear reduced-order model which estimates the time advancement of POD modes was constructed by the least squares method based on the time histories of the POD modes. It has been revealed that the model reproduces the original POD modes well in the beginning. However, the reproducibility of the model has not yet to investigate quantitatively.

In this study, we propose the quantitative evaluation method of the model performance based on the estimation error and investigate the effects of parameters for the construction of the model on the reproducibility. Additionally, the effect of the flow configuration is also evaluated by comparing the estimation results on different angle of attacks. In this paper, we explain the effect of the angle of attack due to space limitation.

2 Quantitative Evaluation of the Linear Reduced-order Model

The linear reduced-order model is constructed based on POD modes obtained from the PIV data. POD modes are acquired from the data matrix $X$ which consists of the two-dimensional velocity components $u$ and $v$. In the construction of the present model, the amplitude and temporal mode of the first $r$ POD modes $z(n) = [z_1(n) \ z_2(n) \ ... \ z_r(n)]^T$ are used. $r$ is the number of POD modes for constructing the reduced-order model. We have defined the model equation as follows [3]:

$$z(n) = A z(n-1).$$  (1)

The linear coefficient matrix $A$ is computed by the least squares method using the set of $z$. The estimation of the time advancement of POD modes was conducted recursively using the original data at the initial time step ($n = 1$). Thus, the estimated POD modes at the $n$th time step are acquired as follows:

$$\hat{z}(n) = A^{n-1} z(1).$$  (2)

We focus on the estimation error of the model in order to evaluate the reproducibility of the model quantitatively. Equation (2) explains that many estimation results are obtained by changing the original data for the model as the initial condition: $\hat{z}(n)(p) = A^{n-1} z(p)$. We define the instantaneous error of the model at each time step as follows:
\[ e^{(p)}(n) = \sqrt{\sum_{k=1}^{N} (z^k(n) - \hat{z}^{(p)}(n))^2} = \sqrt{\sum_{k=1}^{N} (e^k(n))^2}, \]  

(3)

and take the ensemble averaging of them in order to depict the smooth estimation-error curve:

\[ e(n) = \frac{1}{q} \sum_{p=1}^{q} e^{(p)}(n) . \]  

(4)

In this study, we acquire the “permissive time range” \( n_{perm} \) as the reproducibility of the model. \( n_{perm} \) satisfies the following equation:

\[ e(n_{perm}) = 0.632 \sigma_{RMS} = 0.632 \sqrt{\frac{1}{N} \sum_{n=1}^{N} \sum_{k=1}^{N} (z^k(n))^2} . \]  

(5)

3 Results

Figure 1 shows the effect of angle of attack on the model reproducibility on the condition of \( r = 10 \). The vertical axis represents the permissive time range non-dimensionalized by the freestream velocity \( U \) and the chord length of the airfoil \( c \). It corresponds to the model reproducibility. The horizontal axis represents the angle of attack \( \alpha \). This result illustrates that the reproducibility gets better in the case of a higher angle of attack. It implies that the linear system can express the time advancement of larger flow structure better because the size of the vortex structure owing to flow separation becomes larger as the angle of attack increases.

![Fig. 1 Relationship between the angle of attack and the model reproducibility (r = 10)](image)

4 Conclusion

In this study, a quantitative evaluation method for the reproducibility of the linear reduced-order model is introduced and the effects of various parameters on the model performance are investigated. The comparison with the reproducibility on the different conditions of the angle of attack shows that the reproducibility depends on the flow configuration. The large-scale flow structures seem to be reproduced well by the present linear model. In addition, we will show the effects of other parameters (the type of coefficient matrix \( A \) and the number \( r \) of POD modes for modeling) in the presentation.

References

