Enhancing PIV with statistical methods

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Abstract Particle Image Velocimetry (PIV) is a well-established technique for quantitative flow-field measurements in a wide range of applications [1]. Recent assessments [2] have shown that the technique has matured performances in terms of temporal and spatial resolution which open interesting pathways for fundamental and applied research. The main bounds of the techniques are usually expressed in terms of Dynamic Velocity Range (DVR) and Dynamic Spatial Range (DSR). The DVR is the ratio between the largest and the smallest measurable velocity (i.e. the velocity uncertainty). The DSR is defined as the ratio between the largest and the smallest resolved scale. The smallest resolvable scale is approximately the interrogation spot size in PIV, or of the order of the average distance between particles in tracking algorithms. PIV and Particle Tracking Velocimetry (PTV) theoretical developments have for long pushed towards increasing the DVR, i.e. reducing the uncertainty of the velocity also at the smallest scales, thus leading to full exploitation of the DSR. Nowadays, both techniques are mature enough to guarantee low uncertainty also at the smallest resolved scales, thus setting the DSR as the real limiting factor to obtain fully-resolved measurements of turbulent flows at high Reynolds number. Considering the current trend towards PTV (especially in 3D [4]), it is not foreseeable to push the smallest scale below the average particle distance in large-Reynolds-number turbulent-flow measurements, where normally time resolution is normally not achieved due to hardware limits, thus preventing the use of solutions to pour time-resolution into space.

The resolution of turbulent statistics is normally improved using ensemble approaches [5][6][7], in which PTV is carried out on each image pair, and the vectors extracted are stored in an ensemble. The ensemble is then spatially divided in bins, and statistics are computed onto the vectors of the ensemble included in each bin. The achieved resolution depends on the desired uncertainty level and on the number of snapshot. Agüera et al. [7] have demonstrated that a substantial improvement of the spatial resolution can be achieved performing a spatial polynomial fitting of the velocity vectors inside each bin, thus reducing spurious effects of velocity gradients.

In this talk, a data-driven approach to increase the spatial resolution of instantaneous measurements will be shown, following the ensemble principle exploited for the statistics. The method puts its basis on Proper Orthogonal Decomposition (POD). POD is widely used to post-process PIV data, with quite straightforward application when data are available on a grid which does not change in time. This leads directly to the decomposition $U = \Psi\Sigma\Phi^{\dagger}$, where U is the snapshot matrix, Ψ contains the temporal modes, Σ is a diagonal matrix containing the singular values and Φ contains the spatial modes (defined on the same grid points of the PIV data). In PTV, data are available on a scattered grid, with the position of the data depending on the particles position in each snapshot. The standard approach is to interpolate the data on a regular grid, and then perform the POD analysis. This limits the grid resolution to the interspacing between particles to avoid interpolation artifacts. The idea proposed here is to compute the spatial modes using the relation $\Psi^{\dagger}_{PIV}\hat{U} = \Sigma_{HR}\Phi^{\dagger}_{HR}$. The temporal modes Ψ_{PIV} are directly extracted from the results of a standard cross-correlation analysis of the images. The field $\hat{U}$ is built by binning the scattered measurement field and setting the bin velocity values with a simple nearest neighbour approach. Bins without particles inside in a certain snapshot will be assigned zero value and will not contribute in building up the spatial modes. The matrices Σ_{HR} and Φ_{HR} contain the singular values and the spatial modes corresponding to the selected bin resolution. The entries of the matrix $\Sigma_{HR}\Phi^{\dagger}_{HR}$ are computed directly using the previous relation and corrected by weighted according to the number of occurrences of non-zero contributions from the product $\Psi^{\dagger}_{PIV}\hat{U}$. With this approach, one can in principle shrink the size of the bin (provided that a sufficiently large ensemble is available) and reconstruct the individual fields using $U_{HR} = \Psi_{PIV}\Sigma_{HR}\Phi^{\dagger}_{HR}$.

An application of this principle to the wake of a fluidic pinball is shown in Fig. 1. The fluidic pinball [9][10] is a set of three circular cylinders of diameter D immersed in a viscous incompressible uniform freestream at speed U_{∞}. A dataset from simulations at $Re_D = 130$ (i.e. in the chaotic regime [10]) is used to produce virtual images, with randomly generated particles displaced according to the local velocity field. A set of 1000 images has been generated with resolution of 50 pixels/D and particle image density of 0.03 particles per pixels. The PIV analysis has been carried out with a 40×40 pixels interrogation window. The high-resolution POD analysis has been carried out on bins of approximately 4×4 pixels, containing approximately 500 particles over the ensemble. The results in Fig. 1 report an example of instantaneous realization with the standard PIV analysis, the high-resolution reconstruction, the exact field and the corresponding error fields of PIV and high-resolution PTV with respect to the exact field.
Keywords: Particle Image Velocimetry, Proper Orthogonal Decomposition, spatial resolution

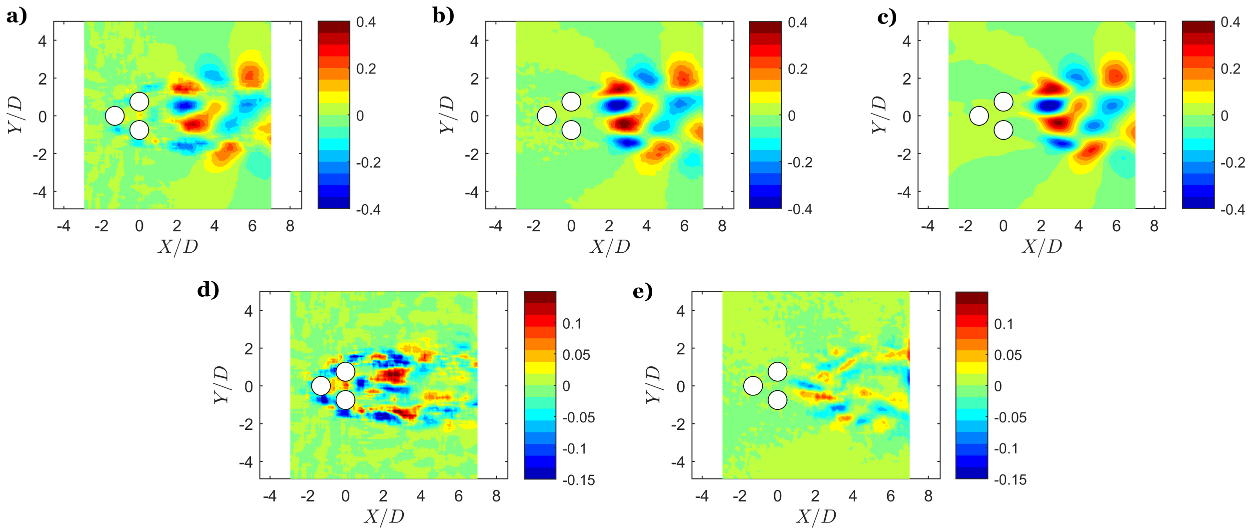

Fig. 1 Instantaneous fluctuations of the streamwise velocity component and error contours of the fluidic pinball testcase: **a)** PIV field **b)** high-resolution PTV **c)** reference exact field **d)** error of PIV **e)** error of high-resolution PTV.

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