Optical flow analysis of fluctuating motions : flow of dense granular suspensions and drying of hydrogels

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Abstract An innovative trajectory method was developed to analyze the fluctuating motions of photometric patterns in an image sequence to efficiently extract both the 2d displacement field and the diffusion coefficient in the motion direction. Two different experimental situations are investigated, namely the flow of dense granular suspensions in a Couette cell and the drying kinetics of hydrogels.

Keywords: Optical flow, velocimetry, fluctuating motions, dense granular suspension, gel drying

1 Introduction

Classical flow visualization methods as Particle Image Velocimetry (PIV) or Particle Tracking Velocimetry (PTV) are based on the object recognizing principle [1]. Cross correlation methods are widely used to determine the average motion of small groups of tracer particles within sub-regions. However, one of the main drawbacks of Digital PIV methods is the inability to accurately resolve random particle displacements and velocity gradients that lead to a loss of correlation and the occurrence of false vectors [2]. Yet, optical flow methods now widely used in computer vision and robotics offer a powerful way to extract a dense velocity field from the observation of photometric patterns in an image sequence [3][4]. However, the mainstream of research in fluid mechanics only gradually evolves towards motion estimation in space-time windows [5]. An innovative trajectory method is presented to analyse locally stationary linear or curvilinear motions of photometric patterns and applied to the study of the non linear flow features of dense granular suspensions in a Couette cell or the drying kinetics of hydrogels cast in a plate.

2 Trajectory method

Image sequences of the uniform or fluctuating vertical motion of disks (diameter d, displacement δy per frame, rms vertical fluctuation $<\delta y^2>^{1/2}$) were generated through Monte Carlo simulations (Fig.1a). A uniform motion of disks with the same velocity results in a textured spatiotemporal window $T(y,t)$ with similarly oriented trajectories (Fig. 1b). The autocorrelation image $\mathrm{A}[T]$ shows a centered ridge line whose tilt angle ψ gives the disk displacement per frame $\delta y = (\omega_t / \omega_s)\,\mathrm{tg}\,\psi$ with $\omega_t = 1\,\text{pixel/frame}$ and $\omega_s = 1$.

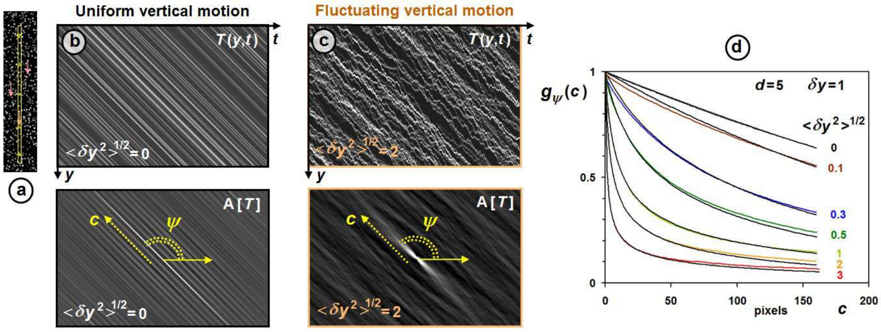

Fig. 1 (a) Image sequence of vertically moving disks (diameter d = 5 pixels, δy = 1 pixel/frame, surface density Φ=20%) and horizontal spatiotemporal projection of grey levels within a yellow rectangular ROI (Region Of Interest). Space - time windows $T(y,t)$ and centered autocorrelation images $\mathrm{A}[T]$ for either uniform (b) or fluctuating vertical motions of disks (c). (d) Normalized correlation function $g_\psi(c)$ along the tilt angle ψ of the ridge line in the autocorrelation image as a function of the rms disk fluctuation $<\delta y^2>^{1/2}$ (simulations and theoretical predictions).

Random fluctuations of the disks' positions along the direction of motion further induce a loss of correlation along the ridge line in the autocorrelation image (Fig.1c). Interestingly, the decrease of the grey level intensity along the ridge line obeys a power law $g_{\psi}(c) \approx c^{-1/2}$ (colored solid lines in Fig.1d). As a helpful analogy, the 3d problem of the intensity fluctuations in fluorescence spectroscopy correlation (FCS) [6] gives the functional form of the correlation signal $g_{\psi}(c) \approx (1 + c\cos\psi / c_d)^{-1/2}$ where c_d is a spatiotemporal length for a mean square displacement $d^2 = 2 < \delta y^2 > c_d$ that shifts the disk out of the ideal straight path in the spatiotemporal window. The above relation nicely accounts for the non linear decay of $g_{\psi}(c)$ along the ridge line (black solid lines in Fig.1d). The robustness of the trajectory method was investigated with respect to the radius, the density and the brightness distribution of disks or possible out of plane motion, giving an estimated displacement δy_{est} per frame with a precision of about 2% insofar as $< \delta y^2 >^{1/2} / \delta y < 3$ and an estimated rms fluctuation $< \delta y^2 >^{1/2}_{\text{est}}$ with a precision less than 10% for $< \delta y^2 >^{1/2}$ as large as $10\,\delta y$.

3 Dense granular suspensions in a Couette flow

The flow of a 55% dense granular suspension made of acrylate (PMMA) spheres (diameter d = 200µm) immersed in a fluorescent fluid matching the refractive index of particles was investigated in a Couette cell (Fig.2a). Particles are heavier than the fluid ($\delta\rho = 0.18\,\text{g/cm}^3$) but a mixture of liquids may provide a precise density-matching ($\delta\rho = 0$). An horizontal laser sheet allows the tracking of photometric patterns moving along circular paths. A transformation of discrete polar coordinates $[\theta(t), r(t)]$ into discrete Cartesian coordinates $(\theta r, t)$ is therefore used to construct the spatiotemporal window $T(\theta r, t)$ (Fig.2b and 2c). The local azimuthal velocity $V_{\theta}(r) = (\omega_t / \omega_s) \tan\psi(r)$ and the rms angular fluctuation $< \delta\theta^2 >^{1/2}$ are estimated from both the tilt angle $\psi(r)$ and the correlation signal $g_{\psi}(c, r)$ along the ridge line in the autocorrelation image A$[T]$. Finally, the trajectory method nicely resolves both slip phenomena and the non linear features of the velocity field as a result of microscopic particle migration in non homogeneous flows [7].

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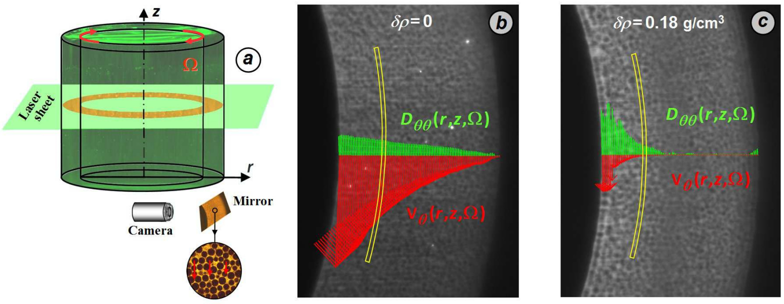

Fig. 2 (a) Experimental setup for the observation of the motion of fluorescent patterns in a refractive index - matched suspension. Azimuthal velocity $V_{\theta}(r)$ (red arrows) and self diffusion coefficient $D_{\theta\theta}(r)$ (green arrows) of particles in a 55% density-matched (b) or a 55% non iso-dense suspension (c) ($\Omega = 0.114$rad/s, z = 10mm and 15mm < r < 20mm.)

4 Drying kinetics of hydrogels

The drying kinetics of a 1.5% agar gel cast in a plate was monitored when observing from the top the 2d motion $\delta e(r,t)$ of seed particles on the free surface of the thinning gel (Fig. 3a) [8]. Perspective effects result in an apparent horizontal displacement $\delta e^*(r,t)$ of seed particles in the image sequence. For purely vertical motions of seed particles, the spatiotemporal analysis of an image sequence gives evidence of a centripetal apparent motion $\delta e^*(r,t)$ of seed particles (Fig. 3b) and a uniform gel thinning rate of about 18nm/s after only 1 hour observation of the gel. However, the displacement field of seed particles may exhibit a radial asymmetry (Fig. 3c) because of the gel sliding on the solid surface leading to the long term detachment of the gel from the lateral walls of the plate. The trajectory method thus provides a quick and powerful tool to test the stability of cellular culture media in Petri dishes when incubated and left dry at constant temperature.

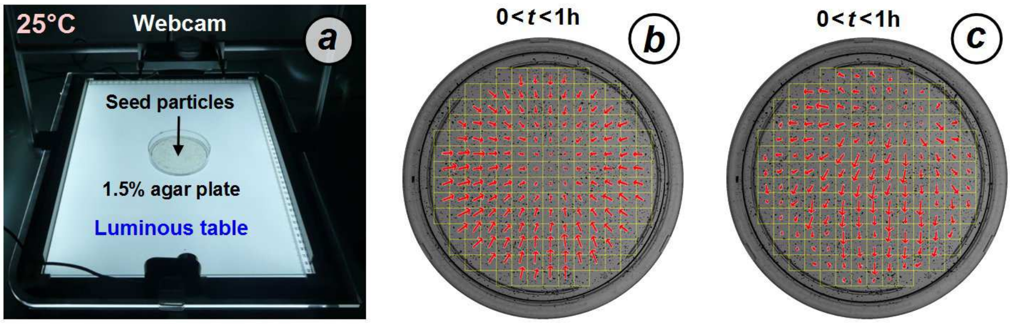

Fig. 3 (a) Experimental setup for the observation of the drying kinetics of a 1.5% agar gel plate. Apparent displacement field $\delta e^*(r,t)$ of seed particles displaying either centripetal (b) or convective motions (c) after only 1 hour observation of the gel.

5 Conclusion

The trajectory method appears as a powerful tool to detect and quantify both first and second order motions of fluctuating patterns. Surprisingly, large fluctuations remain precisely measurable despite lower accuracy in the estimate of the average displacement of the pattern which interrogates about the ability of the primary visual cortex of mammals to detect random displacements of preys.

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